Technical Notes

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A Nonlinear Solution for Parachute Suspension Line Deformation

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Statement of the Problem

An elementary solution for the large, static deformations of a generic suspension line in a solid, flat, circular parachute under the action of a force distribution similar to that transmitted from the adjacent fabric is described herein.

We idealize the suspension line as a flexible string, initially unstressed, straight, and occupying the region $Y=0$, $0 \leqq X \leqq L$. Its deformation is defined (see Fig. 1) by

$$x=x(X), \qquad y=y(X)$$

i.e., (x,y) are the deformed coordinates of the particle that initially has coordinates $(X,0)$. $\theta(X)$ is the deformed slope angle of the string element initially at X. The governing equations are obtained as one-dimensional versions of those for membranes, given, for example, by Otto[1]:

$$\tan\theta = \frac{dy}{dx} \tag{1}$$

$$\epsilon \equiv \text{strain} = \left\{ \left(\frac{dx}{dX}\right)^2 + \left(\frac{dy}{dX}\right)^2 \right\}^{1/2} - 1 \tag{2}$$

$$\frac{d\theta}{dX} + (1+\epsilon)p/T = 0 \tag{3}$$

where T, the tensile force, and ϵ are independent of X and assumed to obey the elastic constitutive law

$$T = E^*\epsilon \tag{4}$$

The normal force resultant p, is assumed to be a rectangular pulse in X,

$$\begin{aligned} p &= p_o \qquad \text{in } R = \{X\colon X_V \leqq X \leqq X_E\} \\ &= 0 \qquad \text{elsewhere} \end{aligned} \tag{5}$$

where subscripts V and E denote the inner edge of the vent (the opening at the top of the canopy) and the forward edge of the pressurized region R, as shown in Fig. 2.

Figure 3 illustrates the relation between p and p_f, the fluid pressure on the fabric. Figure 3a shows that $2y\sin(\beta/2)$ is the distance between corresponding points on adjacent lines, i.e., the distance C-D in section A-A. The fluid pressure on the fabric must be in equilibrium with that on line C-D, therefore, the total normal force transmitted by the adjacent fabric to the line at this cross section is

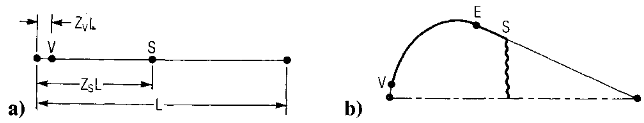

Fig. 1 Deformed line: a) geometry, b) forces.

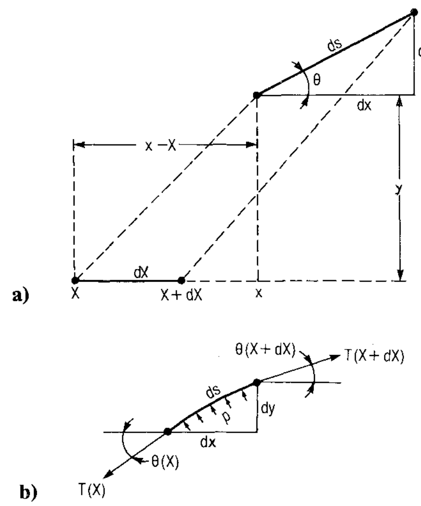

Fig. 2 Line configurations: a) initial, b) during deformation.

$$p_o = 2p_f y\sin(\beta/2) \tag{6}$$

where $\beta = 2\pi/N_L$, and N_L is the number of lines.

Combining this with Eqs. (1-3) and (5), eliminating y, and defining $z = X/L$, we obtain the differential equation for the nonlinear pendulum,

$$\frac{d^2\theta}{dz^2} + \lambda^2\sin\theta = 0$$

$$\begin{aligned} \lambda^2 = G^2 &\equiv \{2p_f L^2(1+\epsilon)^2\sin(\beta/2)\}/T \quad \text{in} \quad R \\ &= 0 \quad \text{elsewhere} \end{aligned} \tag{7}$$

The boundary conditions are

$$\theta = \pi/2, \qquad x=y=0 \text{ at } z=0; \qquad y=0 \text{ at } z=1$$

where $z=1$, $X=L$, is the confluence point of the lines.

Solution

In the unpressurized region $0 \leq z \leq z_V$, the solution is

$$\theta = \pi/2, \qquad y = L(1+\epsilon)z, \qquad x=0$$

and in the unpressurized region $z_E \leqq z \leqq 1$, the solution is

$$\theta = \theta(z_E) = \theta_E, \quad y/L = -(1-z)(1+\epsilon)\sin\theta_E$$

$$x/L = x(z_E)/L + (z - z_E)(1+\epsilon)\cos\theta_E$$

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In R, the solution that joins smoothly with that for $z \leqq z_V$ is given by Abramowitz and Stegun[2] as

$$\sin(\theta/2) = m^{1/2}\mathrm{sn}(\alpha\,|m),\ \ y/L = 2m^{1/2}(1+\epsilon)G^{-1}\mathrm{cn}(\alpha\,|m)$$

$$x/L = 2m^{1/2}G^{-1}\{E(\alpha_V\,|m) - E(\alpha\,|m) - G(z-z_V)\} \tag{8}$$

where

$$m = 1/2 + (Gz_V/2)^2 = \sin^2(\theta_E/2)$$

$$\alpha = A - G(z-z_V), \qquad A = F\{\arcsin(2m)^{-1/2}\,|m\},$$

F and E are incomplete elliptic integrals of the first and second kinds, and sn and cn are Jacobian elliptic functions. If $z_V = 0$, the solution simplifies slightly since $m = 1/2$ and $A = F(\pi/2\,|\,1/2) = K(1/2) = 1.8547$.

To complete the solution, G must be chosen to make y continuous across z_E, which leads to the transcendental equation

$$G(1-z_E)\mathrm{sn}(\alpha_E\,|m)\mathrm{dn}(\alpha_E\,|m) + \mathrm{cn}(\alpha_E\,|m) = 0$$

This is solved numerically for G as a function of z_E and z_V, with results shown in Fig. 4. The empirically fitted function

$$\log G = 0.426 - 0.712q + 0.164q^2, \qquad q = \log z_E$$

is an accurate representation in $z < 0.5$, as long as $z_E - z_V$ is not too small, and is displayed as the solid line in Fig. 4.

Once G is known, we can find T and ϵ. If the string is assumed inextensional, $\epsilon = 0$, and Eq. (7) implies

$$T = T_i \equiv 2p_f L^2 \sin(\beta/2)G^{-2}$$

For a linearly elastic material, the constitutive law [Eq. (4)] leads to a quadratic equation for ϵ with the solution

$$\epsilon = H - (H^2-1)^{1/2} \quad \text{and} \quad H = -1 + E^*/(2T_i)$$

T is then found from Eq. (4). In this solution, $T \to T_i$ as $E^* \to \infty$. The total aerodynamic drag on the canopy is $D = TN_L\cos\theta_E$.

Figure 5 shows the dimensionless deformed shapes predicted by Eq. (8) for the case $z_V = 0$, $E^* = 1500$ lb, $D = 200$ lb, and $N_L = 28$ when $z = 0.1$, 0.3, and 0.5. The variables x' and y' in the graph are defined by $x' = y/L$ and $y' = 1 - (x/L)$.

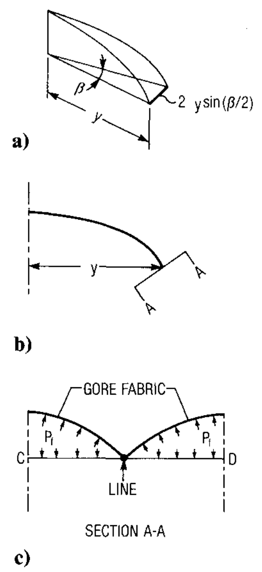

Fig. 3 Fabric and line shape during deformation: a) overall sketch, b) line shape, and c) section A-A.

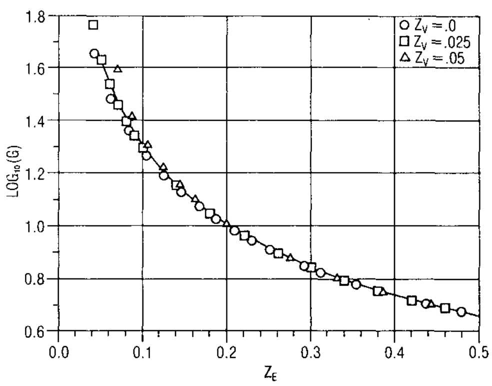

Fig. 4 $\log G$ as a function of z_E.

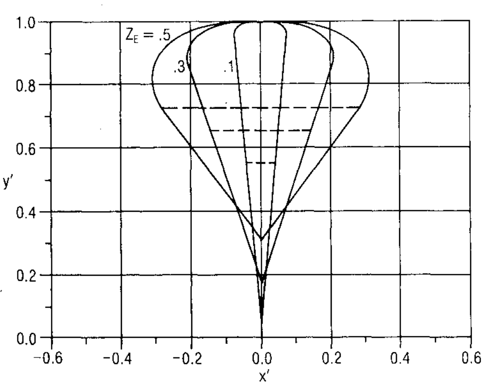

Fig. 5 Profiles of lines for $z_V = 0$ and $z_E = 0.1$, 0.3, 0.5.

Conclusions

The shape of the fully open parachute in Fig. 5 agrees fairly well with the proportions of a typical solid, flat, circular canopy given in Ref. 3 in the parentheses below:

Projected diameter/flat circular diameter = 0.6 (0.67)

Depth/projected diameter = 0.47 (0.41)

In recent years most investigators have used the finite element method combined with hypotheses about the pressure distribution to solve problems in parachute inflation, e.g., Mullins and Reynolds.[4] The family of shapes obtained here may be useful as a check on the correctness of the finite element code for the structural part of the calculation. The neglect of the fabric and line inertia in this solution is not necessarily an impediment to checking, since inertia can also be omitted from the finite element calculation in most cases. Whether the neglect of these inertia forces is too inaccurate in practical calculations is a different question, whose answer depends on the relative magnitudes of these forces and the fabric stress forces acting on the line. These inertia forces were sometimes neglected in earlier studies, e.g., Heinrich and Jamison,[5] but more work may be needed to answer the question definitively.

References

[1]Otto, F., *Tensile Structures*, Vol. 1, M.I.T. Press, Cambridge, MA, 1967, pp. 268-289.

[2]Abramowitz, M. and Stegun, I. A., *Handbook of Mathematical Functions*, National Bureau of Standards, Washington, DC, 1976, pp. 567-626.

[3]"Recovery System Design Guide," AFFDL-TR-78-51, Dec. 1978.

[4]Mullins, W. M. and Reynolds, D. T., Stress Analysis of Spacecraft Parachutes using Finite Elements and Large Deformation Theory," AIAA Paper 70-1195, 1970.

[5]Heinrich, H. G. and Jamison, L. R., "Parachute Stress Analysis During Inflation and at Steady State," *Journal of Aircraft*, Vol. 15, 1978, pp. 100-105.